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Non-extensive statistics effects in quark-gluon plasma and in relativistic heavy-ion collisions

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The presence of memory effects and color long-range forces among the many-parton system in the early stage of heavy-ion collisions can affect the particle statistical behavior at the freeze-out temperature. In this context, we calculate, in the framework of the equilibrium generalized non-extensive thermostatistics, the shape of pion transverse mass spectrum and the value of the transverse momentum correlation function of the pions emitted during the central Pb+Pb collisions and we show that the experimental results is well reproduced assuming very small deviations from the standard statistics.

1. Introduction

Lattice QCD calculations predict that nucleons brought to high enough temperature or pressure will make a transition to a state where the quarks are no longer confined into individual hadrons but dissociate into a plasma of quarks and gluons (QGP).

Such a QGP can be found in: early universe, dense and hot stars, neutron stars and nucleus-nucleus high energy collisions where heavy-ions are accelerated to relativistic energies and made to collide. After collision, a fireball is created, expands, cools, freezes-out into hadrons, photons, leptons that are detected and analysed.

Under the hypothesis that QGP is generated in the early stage of the relativistic collisions, the quantities characterizing the plasma, such as lifetime and damping rate of quasiparticles, are usually calculated within finite temperature perturbative QCD since the interactions among quarks and gluons become weak at small distance or high energy. However, this is rigorously true only at very high temperature (even up to five times the critical temperature) while around the critical temperature (that corresponds approximately to the energy scale of the SPS-Cern experiments) lattice calculations show that there are strong non-perturbative QCD effects and QGP cannot be considered as a weakly interacting plasma [1,2].

In order to understand well the relevance of this point, let us consider the color-Coulombic QGP parameter, which can be expressed as the ratio of the average potential energy to the average kinetic energy of particles [2]

$$\Gamma = \frac{\langle P.E.\rangle}{\langle K.E.\rangle} \approx \left(\frac{4\pi n}{3}\right)^{1/3} \frac{4}{3}\frac{\alpha_s}{T} \,, \qquad (1)$$

In perturbative QCD, $\alpha_s \to 0$, $\Gamma \ll 1$ and the plasma is considered as an ideal gas. At $T = T_c \approx 200$ MeV, $\alpha_s = g^2/4\pi \approx 0.5$, $< r > \approx 1$ fm and we get $\Gamma \approx 2/3 < 1$. Therefore, during hadronization, the plasma parameter lies in an intermediate region typical of a non-ideal plasma: collective and individual effects coexist; memory effects and long-range interactions are present. On the other hand, it is not a true strongly coupled plasma ($\Gamma > 1$), and a suitable approach describing a system at these intermediate conditions is still non available.

Recent progresses in statistical mechanics have shown that the non-extensive statistics, proposed by Tsallis [3], can be considered as the natural generalization of the extensive Boltzmann-Gibbs statistics in presence of long-range forces and/or in irreversible processes related to microscopic long-time memory effects. Since these features are present in the early stage of the collisions, we argue that the non-extensive statistics can be more appropriate than the Boltzmann–Gibbs one in the context of high-energy heavy-ion collisions. Here we investigate how these statistical effects can affect the equilibrium properties of experimental observables such as the transverse momentum spectrum and the fluctuation-correlation measure of pions emitted during the collisions.

2. Transverse momentum spectrum and q-blue shift

The transverse momentum distribution depends on the phase-space distribution and usually an exponential shape is employed to fit the experimental data. This shape is obtained by assuming a purely thermal source with a Boltzmann distribution. High energy deviations from the exponential shape are taken into account by introducing a dynamical effect due to collective transverse flow, also called blue-shift.

Let us consider a different point of view and argue that the deviation from the Boltzmann slope at high $p_\perp$ can be ascribed to the presence of non-extensive statistical effects in the steady state distribution of the particle gas. In this framework, at the first order in $(q-1)$ the transverse mass spectrum can be written as [8]

$$\frac{dN}{m_\perp dm_\perp} = C\, m_\perp \left\{ K_1(z) + \frac{(q-1)}{8} z^2 \; [3\, K_1(z) + K_3(z)] \right\} \,, \tag{2}$$

where $z = m_\perp/T$, $m_\perp = \sqrt{p^2 + m^2}$, K_1 and K_3 are the modified Bessel function of the first and the third order, respectively. In the asymptotic limit, $z \gg 1$, we have

$$\frac{dN}{m_\perp dm_\perp} = D\, \sqrt{m_\perp}\, \exp\left(-z + \frac{q-1}{2}\, z^2 \right) \,, \tag{3}$$

and we may obtain the generalized slope parameter or q-blue shift (if $q > 1$)

$$T_q = T + (q-1)\, m_\perp \,. \tag{4}$$

Let us notice that the slope parameter depends on the detected particle mass and it increases with the energy (if $q > 1$) as it was observed in the experimental results [4].

In Fig. 1, we report the experimental $S + S$ transverse momentum distribution (NA35 data [7]) compared with the purely thermal distribution ($q = 1$) and the one obtained in the framework of Tsallis statistics ($q = 1.038$). We will use, consistently, the same value of q to determine the experimental transverse momenta fluctuations. Similar calculations in the framework of non–extensive statistics have been done in Ref.[9].

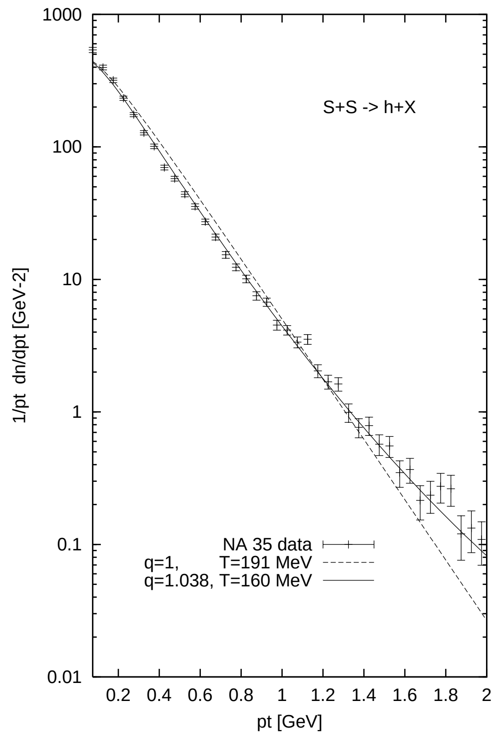

Figure 1. Experimental NA35 [7] transverse momentum distribution compared with the exponential ($q = 1$) thermal distribution (dashed line) and the modified thermal distribution shape (solid line) by using non-extensive statistics ($q = 1.038$)

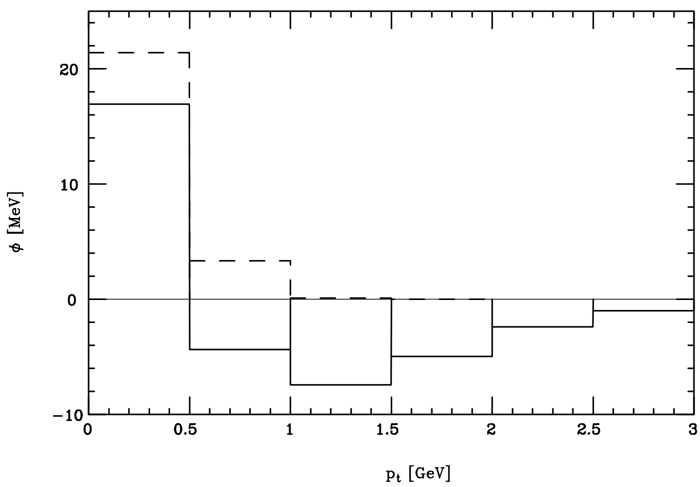

Figure 2. The partial contributions to the correlation measure $\Phi_{p_\perp}$ [MeV] in different $p_\perp$ intervals, at $T = 170$ MeV and $\mu = 60$ MeV. The dashed line refers to standard statistical calculations with $q = 1$, the solid line corresponds to $q = 1.038$.

3. Transverse momentum fluctuations

Gaździcki and Mrówczyński introduced the following quantity [10,11]

$$\Phi_{p_\perp} = \sqrt{\frac{\langle Z^2_{p_\perp} \rangle}{\langle N \rangle}} - \sqrt{\overline{z^2_{p_\perp}}} \,, \tag{5}$$

where $z_{p_\perp} = p_\perp - \overline{p_\perp}$ and $Z_{p_\perp} = \sum_{i=1}^{N} (p_{\perp i} - \overline{p}_{\perp i})$, N is the multiplicity of particles produced in a single event. Non-vanishing Φ implies effective correlations among particles which alter the momentum distribution.

In the framework of non–extensive statistics and keeping in mind that it preserves the whole mathematical structure of the thermodynamical relations, it is easy to show that the two terms in the right hand side of Eq.(5) can be expressed in the following simple form

$$\overline{z^2_{p_\perp}} = \frac{1}{\rho} \int \frac{d^3p}{(2\pi)^3} \left(p_\perp - \overline{p}_\perp\right)^2 \langle n \rangle_q \,, \quad \text{and} \quad \frac{\langle Z^2_{p_\perp} \rangle}{\langle N \rangle} = \frac{1}{\rho} \int \frac{d^3p}{(2\pi)^3} \left(p_\perp - \overline{p}_\perp\right)^2 \langle \Delta n^2 \rangle_q \,, \tag{6}$$

where

$$\overline{p}_\perp = \frac{1}{\rho}\int \frac{d^3p}{(2\pi)^3}\, p_\perp \langle n\rangle_q \qquad \text{with} \qquad \rho = \int \frac{d^3p}{(2\pi)^3}\langle n\rangle_q \; . \tag{7}$$

In the above equations we have indicated with $\langle n\rangle_q$ the following mean occupation number of bosons (valid only for dilute gas and/or small value of q, see Ref.[12] for details)

$$\langle n\rangle_q = \frac{1}{[1+(q-1)\beta(E-\mu)]^{1/(q-1)} \pm 1} \; , \tag{8}$$

and with $\langle \Delta n^2\rangle_q = \langle n^2\rangle_q - \langle n\rangle_q^2$ the generalized particle fluctuations, given by

$$\langle \Delta n^2\rangle_q \equiv \frac{1}{\beta}\frac{\partial \langle n\rangle_q}{\partial \mu} = \frac{\langle n\rangle_q}{1+(q-1)\beta(E-\mu)}\,(1 \mp \langle n\rangle_q) = \langle n\rangle_q^q\,(1 \mp \langle n\rangle_q)^{2-q} . \tag{9}$$

NA49 Collaboration has recently measured the correlation $\Phi_{p_\perp}$ of the pion transverse momentum (Pb+Pb at 158 A GeV) [5] obtaining $\Phi_{p_\perp}^{exp} = (0.6 \pm 1)$ MeV. This value is the sum of two contributions: $\Phi_{p_\perp}^{st} = (5 \pm 1.5)$ MeV, the measure of the statistical two-particle correlation, and $\Phi_{p_\perp}^{tt} = (-4 \pm 0.5)$ MeV, the anti-correlation from limitation in two-track resolution.

Standard statistical calculations ($q = 1$) give [11] $\Phi_{p_\perp}^{st} = 24.7$ MeV at $T = 170$ MeV, $\mu =$ 60 MeV. In the frame of non–extensive statistics, instead, for $q = 1.038$, we obtain the experimental (statistical) value: $\Phi_{p_\perp}^{st} = 5$ MeV at $T = 170$ MeV, $\mu = 60$ MeV.

In Fig. 2, we show the partial contributions to the quantity $\Phi_{p_\perp}$, by using Eq.s (6), and by extending the integration over $p_\perp$ to partial intervals $\Delta p_\perp = 0.5$ GeV at $T = 170$ MeV and $\mu = 60$ MeV. In the standard statistics (dashed line), $\Phi_{p_\perp}$ is always positive and vanishes in the $p_\perp$-intervals above ≈ 1 GeV. In the non-extensive statistics (solid line), instead, the fluctuation measure $\Phi_{p_\perp}$ becomes negative for $p_\perp$ larger than 0.5 GeV and becomes vanishingly small only in $p_\perp$-intervals above ~ 3 GeV. Such a negative value of $\Phi_{p_\perp}$ at high $p_\perp$ could be a clear and unambiguos evidence of the presence of non-extensive regime in heavy-ions collisions.

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